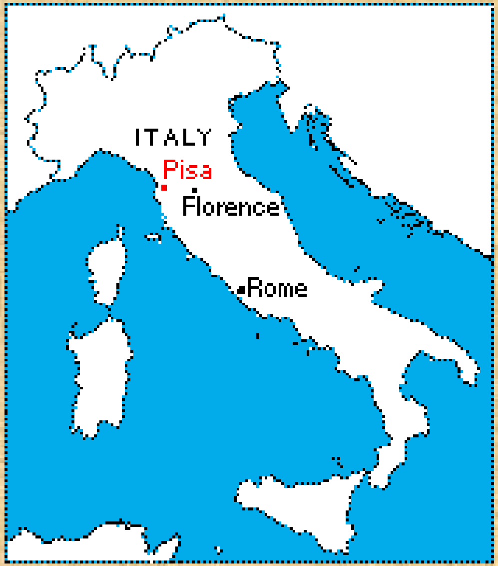
The Fibonacci Numbers
and
The Golden Section

By Zhengyi(Eric) Ge 4th Year Chemical Engineering

Who Was Fibonacci?



~ Born in Pisa, Italy in 1175 AD

~ Full name was Leonardo Pisano

~ Grew up with a North African education under the Moors

~ Traveled extensively around the Mediterranean coast

~ Met with many merchants and learned their systems of arithmetic

~ Realized the advantages of the Hindu-Arabic system

Fibonacci's Mathematical Contributions

~ Introduced the Hindu-Arabic number system into Europe

~ Based on ten digits and a decimal point

~ Europe previously used the Roman number system

~ Consisted of Roman numerals

~ Persuaded mathematicians to use the Hindu-Arabic number system

1 2 3 4 5 6 7 8 9 0 .

Fibonacci’s Mathematical Contributions Continued

~ Wrote five mathematical works

~ Four books and one preserved letter

~ *Liber Abbaci* (*The Book of Calculating*) written in 1202

~ *Practica geometriae* (*Practical Geometry*) written in 1220

~ *Flos* written in 1225

~ *Liber quadratorum* (*The Book of Squares*) written in 1225

~ *A letter to Master Theodorus* written around 1225

The Fibonacci Numbers

~ Were introduced in *The Book of Calculating*

~ Series begins with 0 and 1

~ Next number is found by adding the last two numbers together

~ Number obtained is the next number in the series

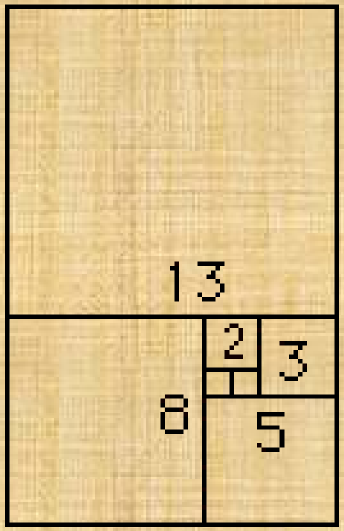
~ Pattern is repeated over and over

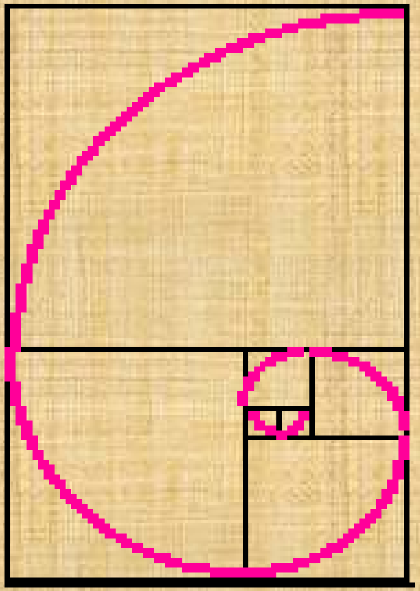
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, …

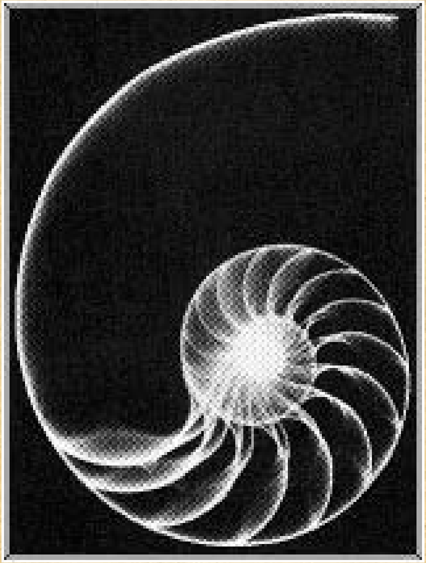
$$F(n + 2) = F(n + 1) + Fn$$

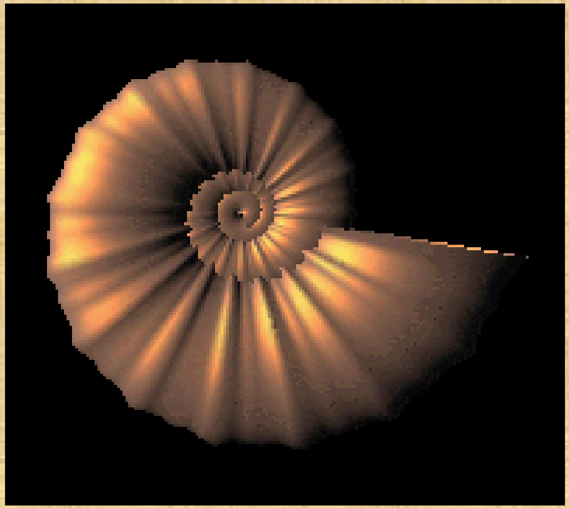
The Fibonacci Numbers in Nature

~ Fibonacci spiral found in both snail and sea shells



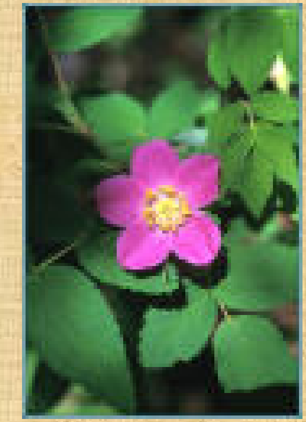






The Fibonacci Numbers in Nature Continued

Lilies and irises = 3 petals



Buttercups and wild roses = 5 petals

Corn marigolds = 13 petals

Black-eyed Susan’s = 21 petals

The Fibonacci Numbers in Nature Continued

~ The Fibonacci numbers can be found in pineapples and bananas

~ Bananas have 3 or 5 flat sides

~ Pineapple scales have Fibonacci spirals in sets of 8, 13, 21

The Golden Section

~ Represented by the Greek letter Phi

~ Phi equals ± 1.6180339887 … and ± 0.6180339887 …

~ Ratio of Phi is 1 : 1.618 or 0.618 : 1

~ Mathematical definition is $\text{Phi}^2 = \text{Phi} + 1$

~ Euclid showed how to find the golden section of a line

< - - - - - - - - 1 - - - - - - - - >

A G B

g $1 - g$

$$\frac{GB}{AG} = \frac{AG}{AB} \quad \text{or} \quad \frac{1 - g}{g} = \frac{g}{1}$$

so that $g^2 = 1 - g$

The Golden Section and The Fibonacci Numbers

~ The Fibonacci numbers arise from the golden section

~ The graph shows a line whose gradient is Phi

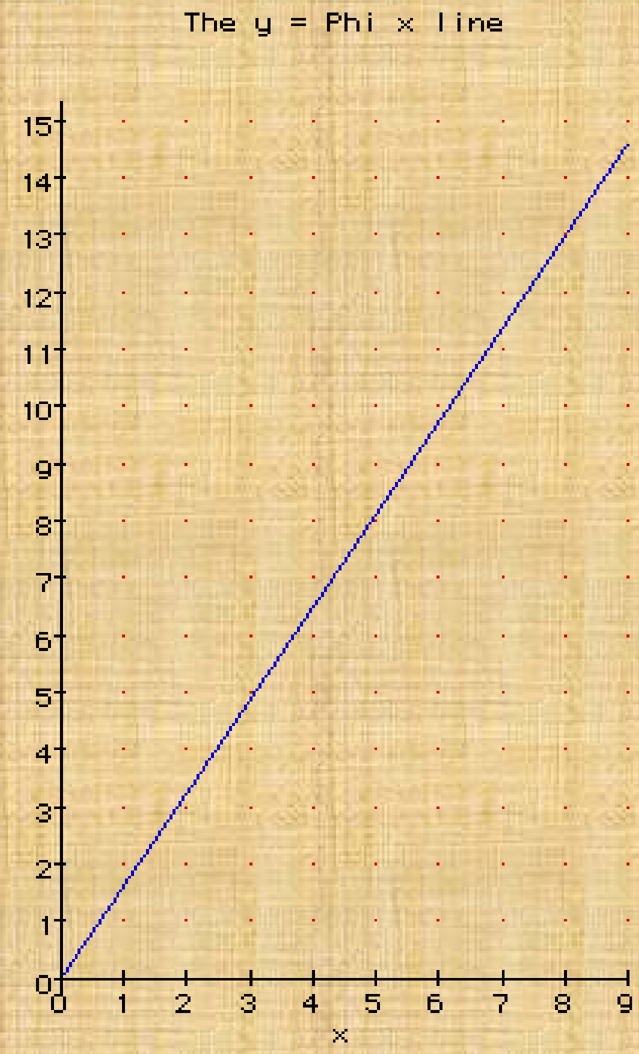
~ First point close to the line is (0, 1)

~ Second point close to the line is (1, 2)

~ Third point close to the line is (2, 3)

~ Fourth point close to the line is (3, 5)

~ The coordinates are successive Fibonacci numbers

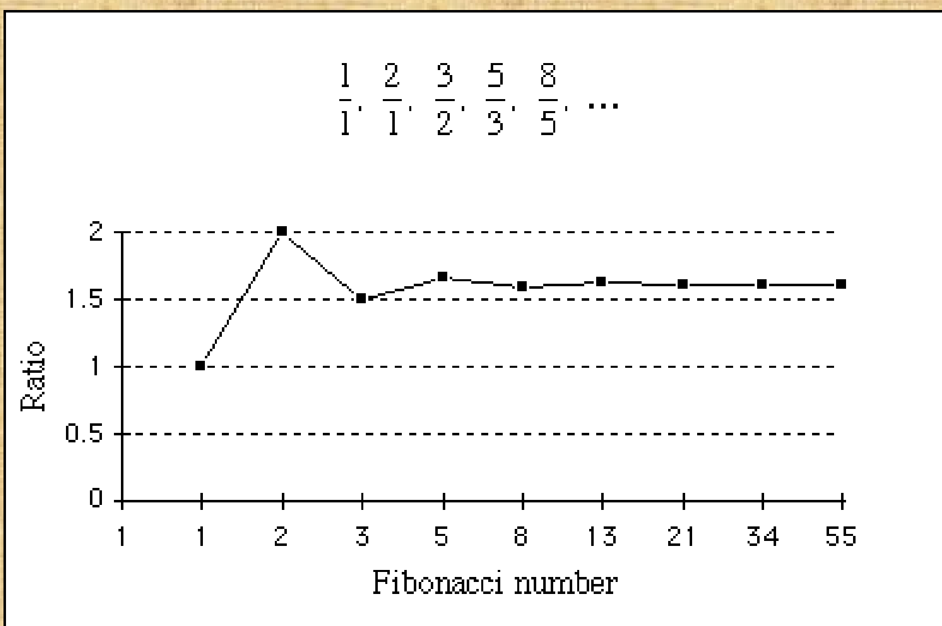


The Golden Section and The Fibonacci Numbers Continued

~ The golden section arises from the Fibonacci numbers

~ Obtained by taking the ratio of successive terms in the Fibonacci series

~ Limit is the positive root of a quadratic equation and is called the golden section



If you take two successive terms of the series, a, b, and $a+b$ then

$$\frac{b}{a} \cong \frac{a+b}{b}$$

$$\cong \frac{a}{b} + 1$$

We define the golden section, ϕ (*phi*), to be the limit of $\frac{b}{a}$, so:

$$\phi = \frac{1}{\phi} + 1$$

$$\phi^2 - \phi - 1 = 0$$

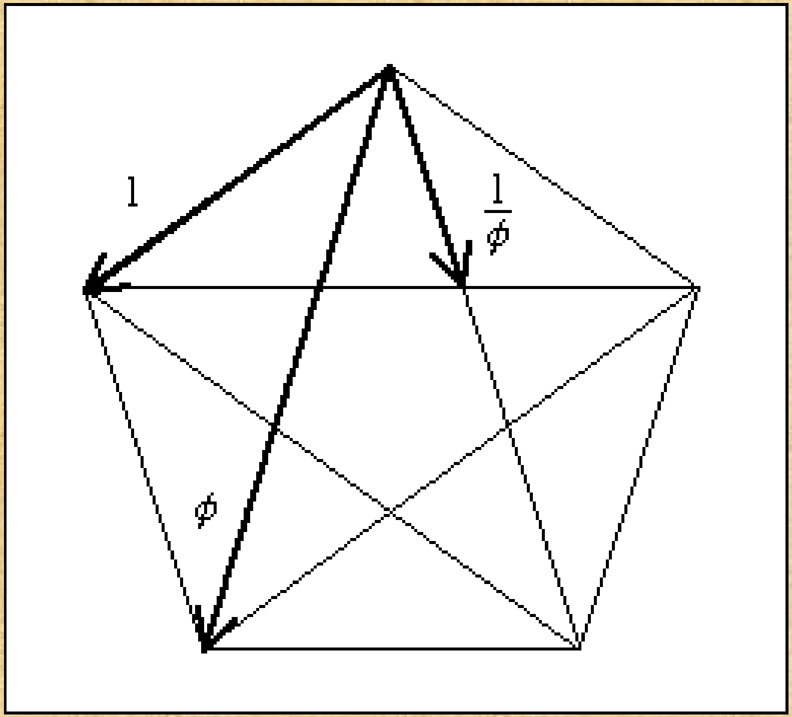
$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

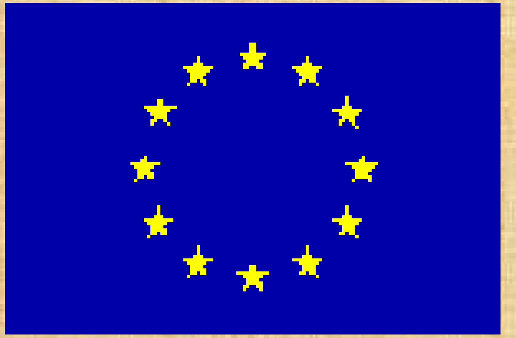
The Golden Section and Geometry

~ Is the ratio of the side of a regular pentagon to its diagonal

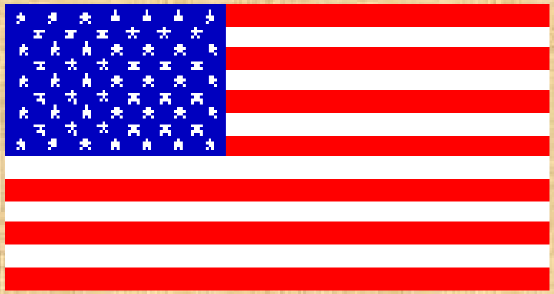
~ The diagonals cut each other with the golden ratio

~ Pentagram describes a star which forms parts of many flags





European Union



United States

- The Golden Proportion is the basis of the *Golden Rectangle*, whose sides are in golden proportion to each other.

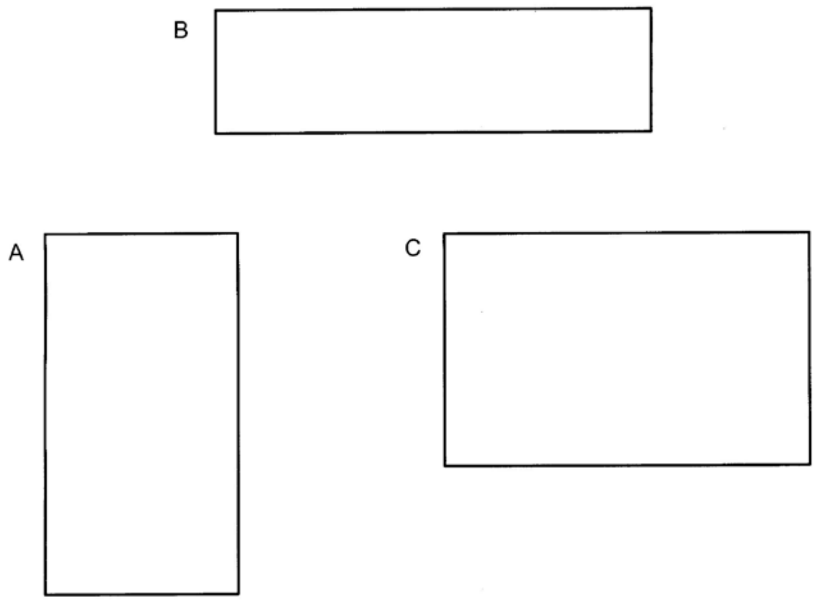
- The Golden Rectangle is considered to be the most visually pleasing of all rectangles.

- For this reason, as well as its practicality, it is used extensively:

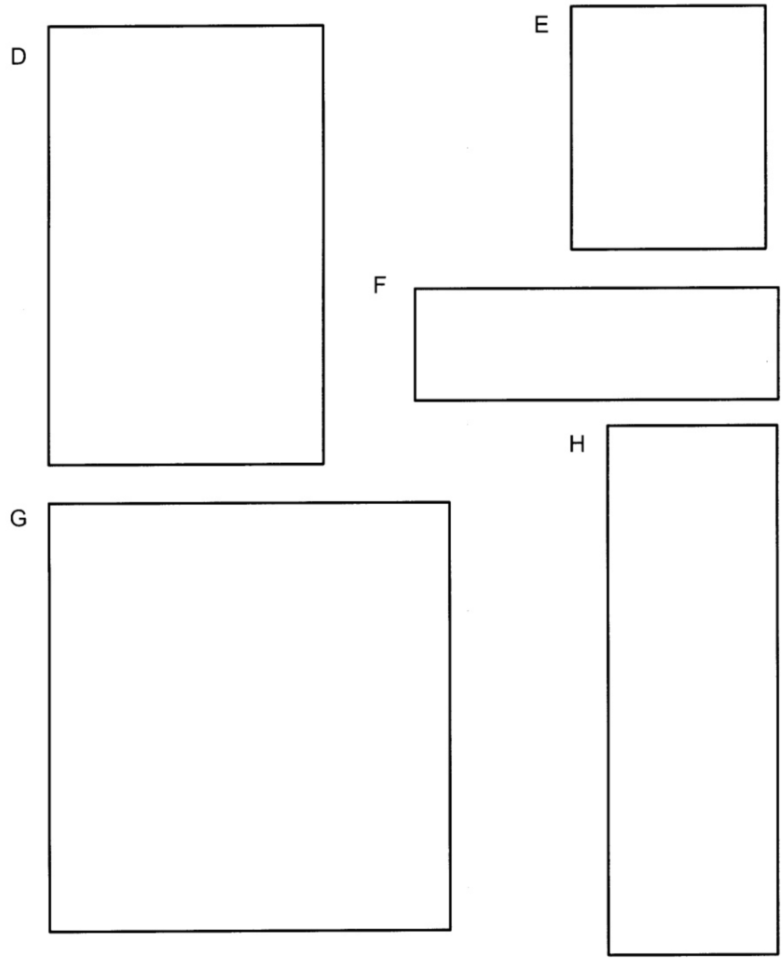
- In all kinds of design, art, architecture, advertising, packaging, and engineering; and can therefore be found readily in everyday objects.

- Quickly look at the rectangular shapes on each slide.
- Chose the one figure on each slide you feel has the most appealing dimensions.
- Make note of this choice.
- Make this choice quickly, without thinking long or hard about it.

Group 1



Group 2



- What was special about these special rectangles?
- Clearly it is not their size.
- It was their proportions.
- The rectangles c and d were probably the rectangles chosen as having the most pleasing shapes.
- Measure the lengths of the sides of these rectangles. Calculate the ratio of the length of the longer side to the length of the shorter side for each rectangles.

- Was it approximately 1.6?
- This ratio approximates the famous Golden Ratio of the ancient Greeks.
- These special rectangles are called Golden Rectangles because the ratio of the length of the longer side to the length of the shorter side is the Golden Ratio.

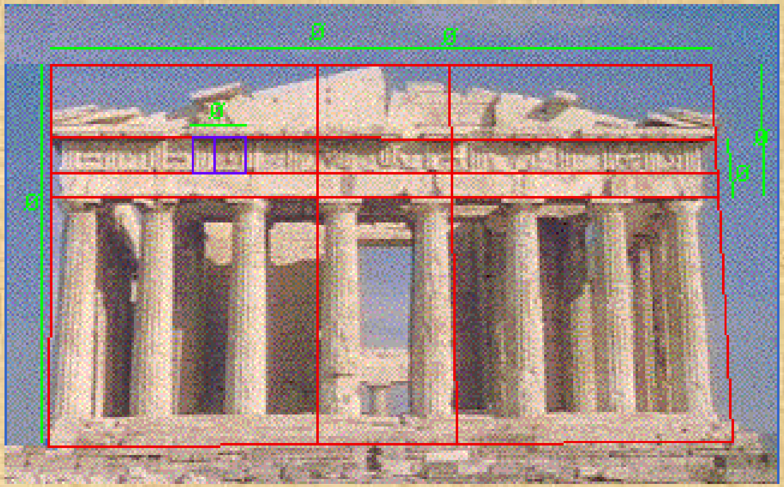
- Golden Rectangles can be found in the shape of playing cards, windows, book covers, file cards, ancient buildings, and modern skyscrapers.
- Many artists have incorporated the Golden Rectangle into their works because of its aesthetic appeal.
- It is believed by some researchers that classical Greek sculptures of the human body were proportioned so that the ratio of the total height to the height of the navel was the Golden Ratio.

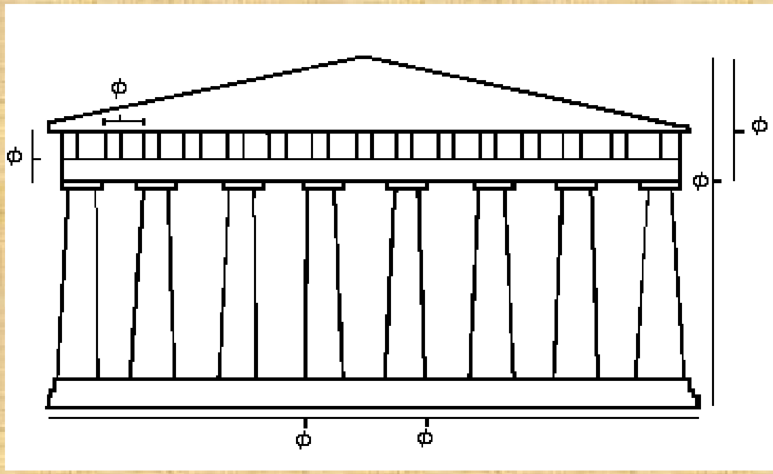
- The ancient Greeks considered the Golden Rectangle to be the most aesthetically pleasing of all rectangular shapes.
- It was used many times in the design of the famous Greek temple, the Parthenon.

The Golden Section in Architecture

~ Golden section appears in many of the proportions of the Parthenon in Greece

~ Front elevation is built on the golden section (0.618 times as wide as it is tall)

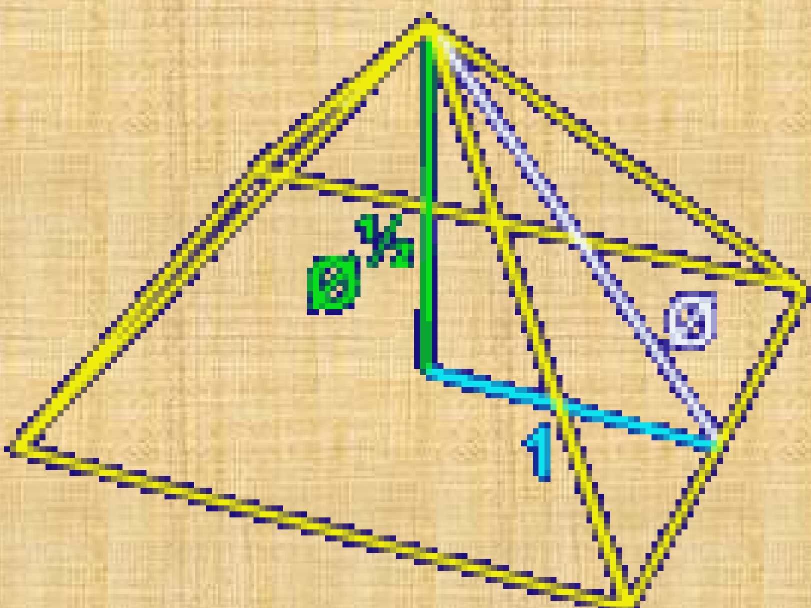




The Golden Section in Architecture Continued

~ Golden section can be found in the Great pyramid in Egypt

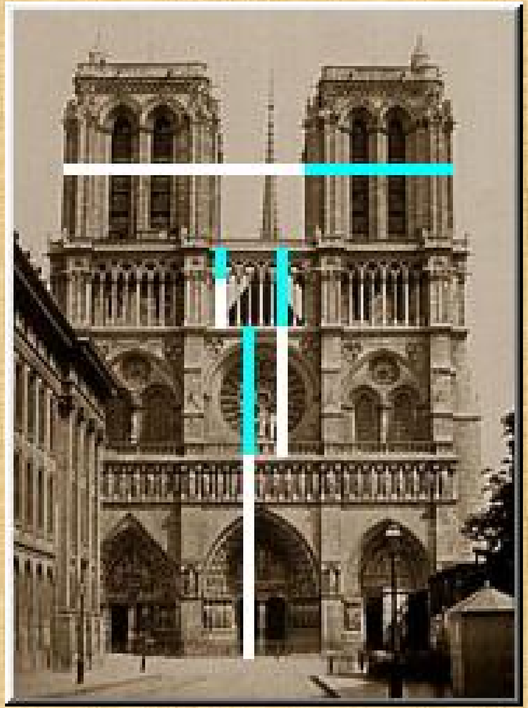
~ Perimeter of the pyramid, divided by twice its vertical height is the value of Phi



The Golden Section in Architecture Continued

~ Golden section can be found in the design of Notre Dame in Paris

~ Golden section continues to be used today in modern architecture



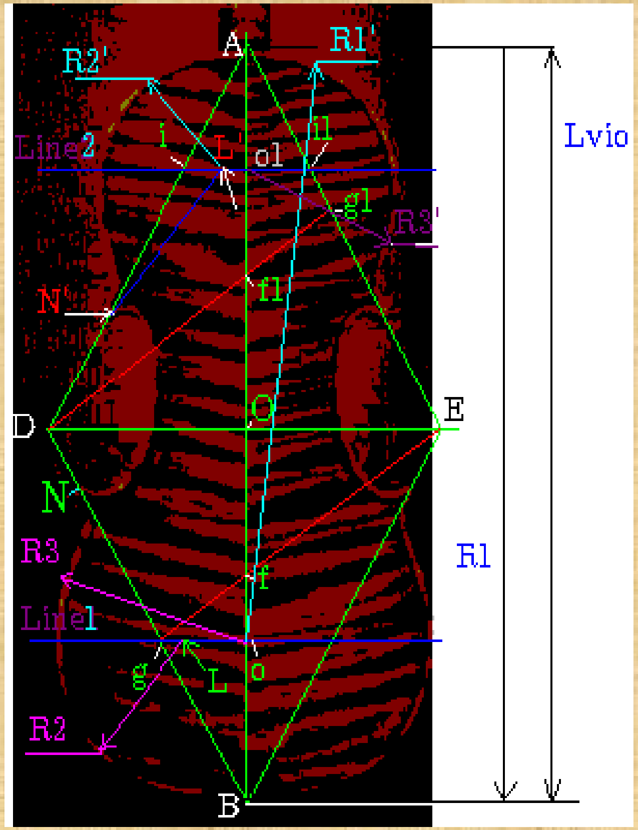
United Nations Headquarters

Secretariat building

The Golden Section in Music

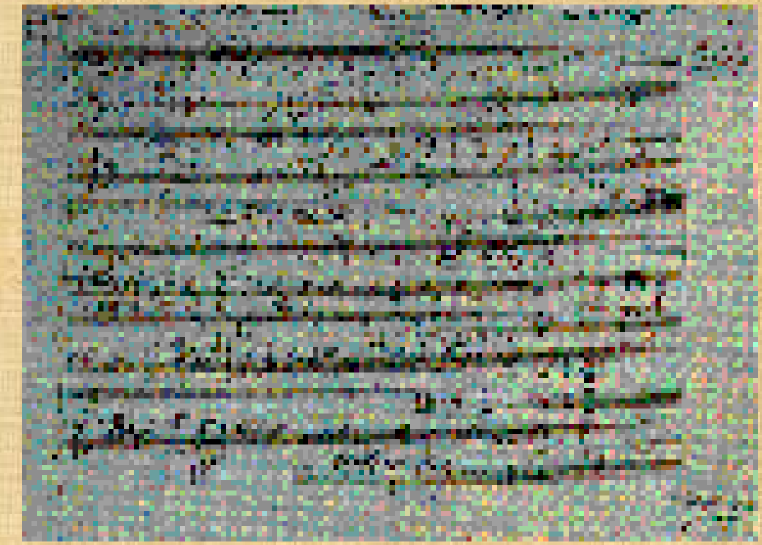
~ Stradivari used the golden section to place the f-holes in his famous violins

~ Baginsky used the golden section to construct the contour and arch of violins

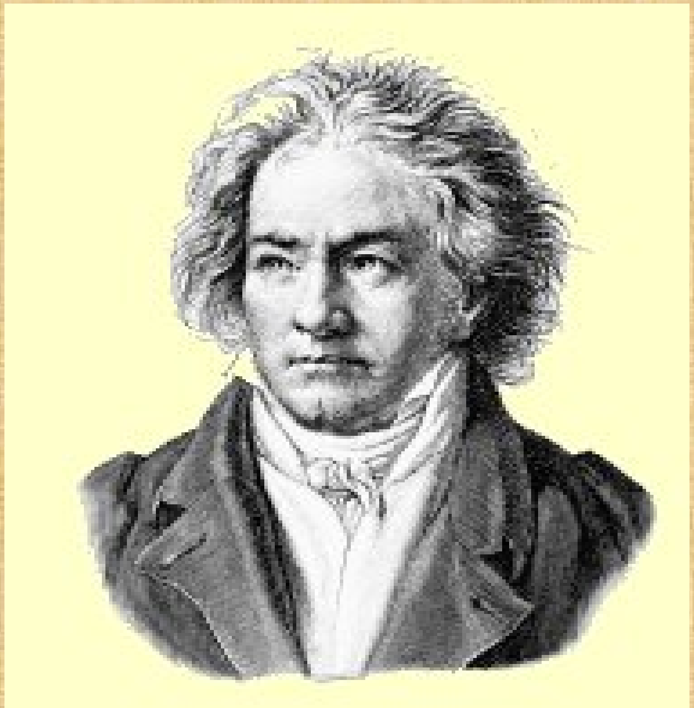


The Golden Section in Music Continued

~ Mozart used the golden section when composing music
~ Divided sonatas according to the golden section
~ Exposition consisted of 38 measures
~ Development and recapitulation consisted of 62 measures
~ Is a perfect division according to the golden section



The Golden Section in Music Continued



~ Beethoven used the golden section in his famous *Fifth Symphony*

~ Opening of the piece appears at the golden section point (0.618)

~ Also appears at the recapitulation, which is Phi of the way through the piece

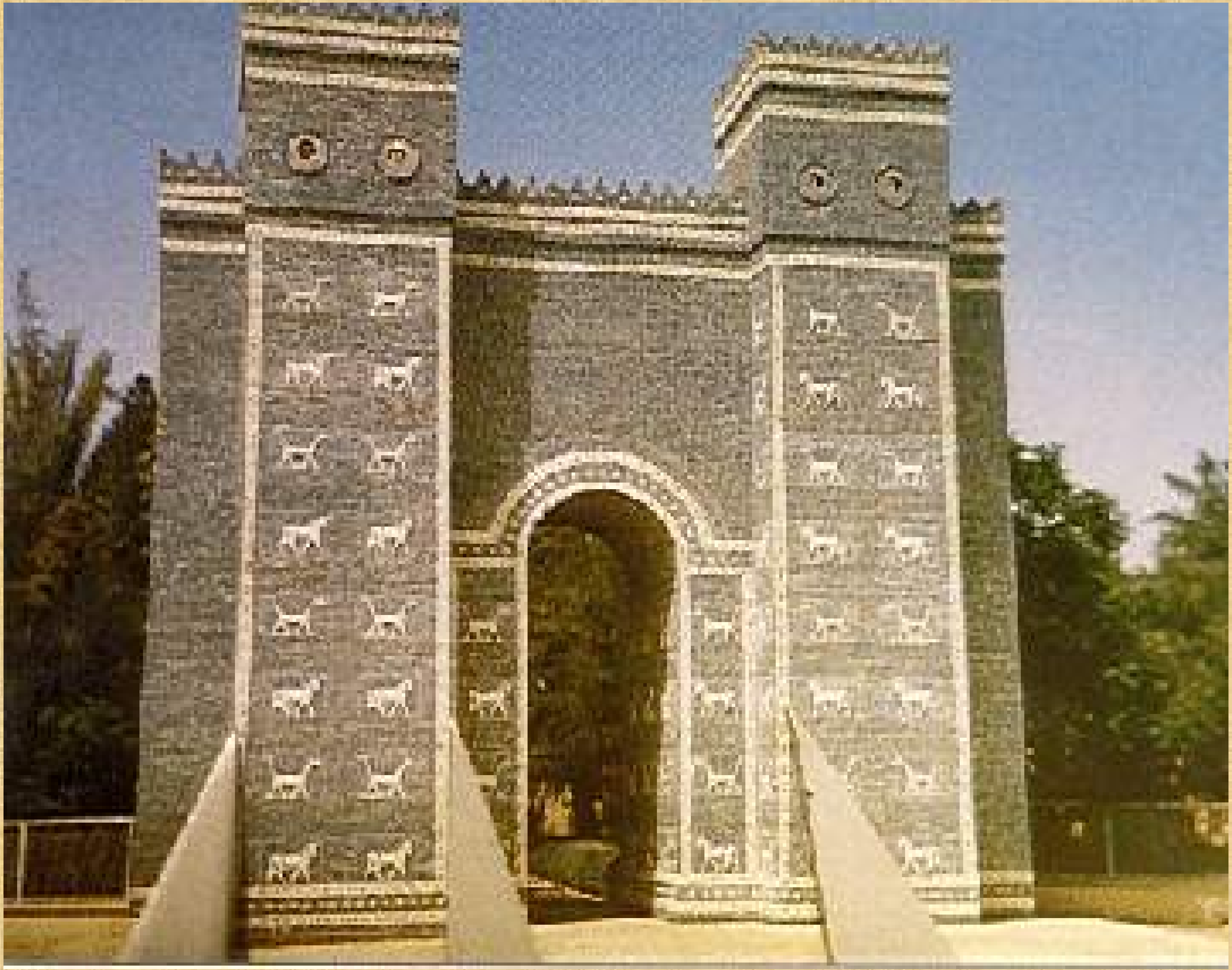
Examples of the Golden Ratio

- On the next pages you will see examples of the Golden Ratio (Proportion)
- Many of them have a gauge, called the Golden Mean Gauge, superimposed over the picture.
- This gauge was developed by Dr. Eddy Levin DDS, for use in dentistry and is now used as the standard for the dental profession.
- The gauge is set so that the two openings will always stay in the Golden Ration as they open and close.

Golden Mean Gauge: Invented by Dr. Eddy Levin DDS

- Dentistry (The reason for the gauge’s creation)…
- The human face…

- Architecture…
- The Automotive industry…
- Music and Musical reproduction…
- Fashion…
- Hand writting…
- General Design…



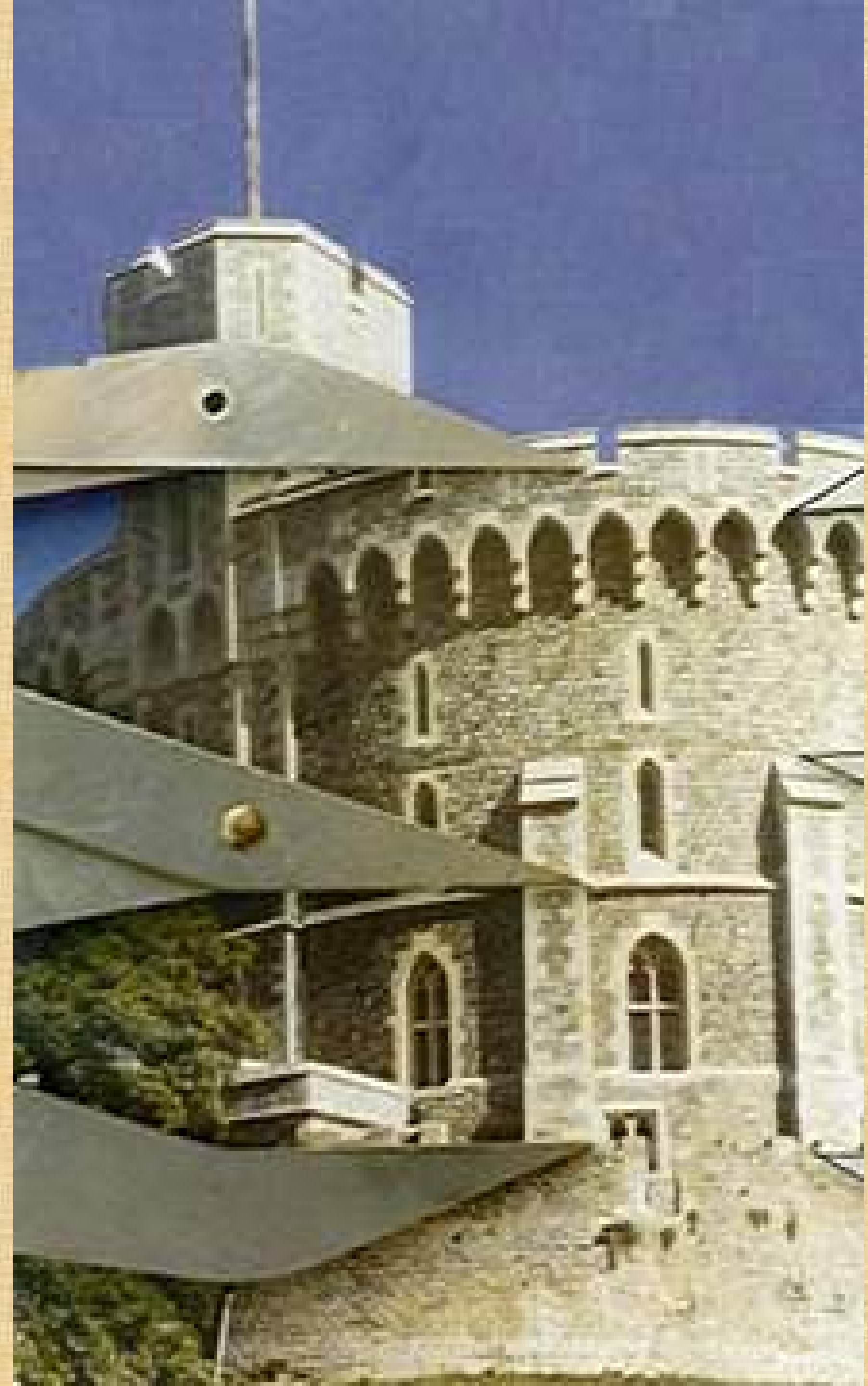
The Bagdad City Gate

Dome of St. Paul: London, England

The Great Wall of China

The Parthenon: Greece

Windson Castle

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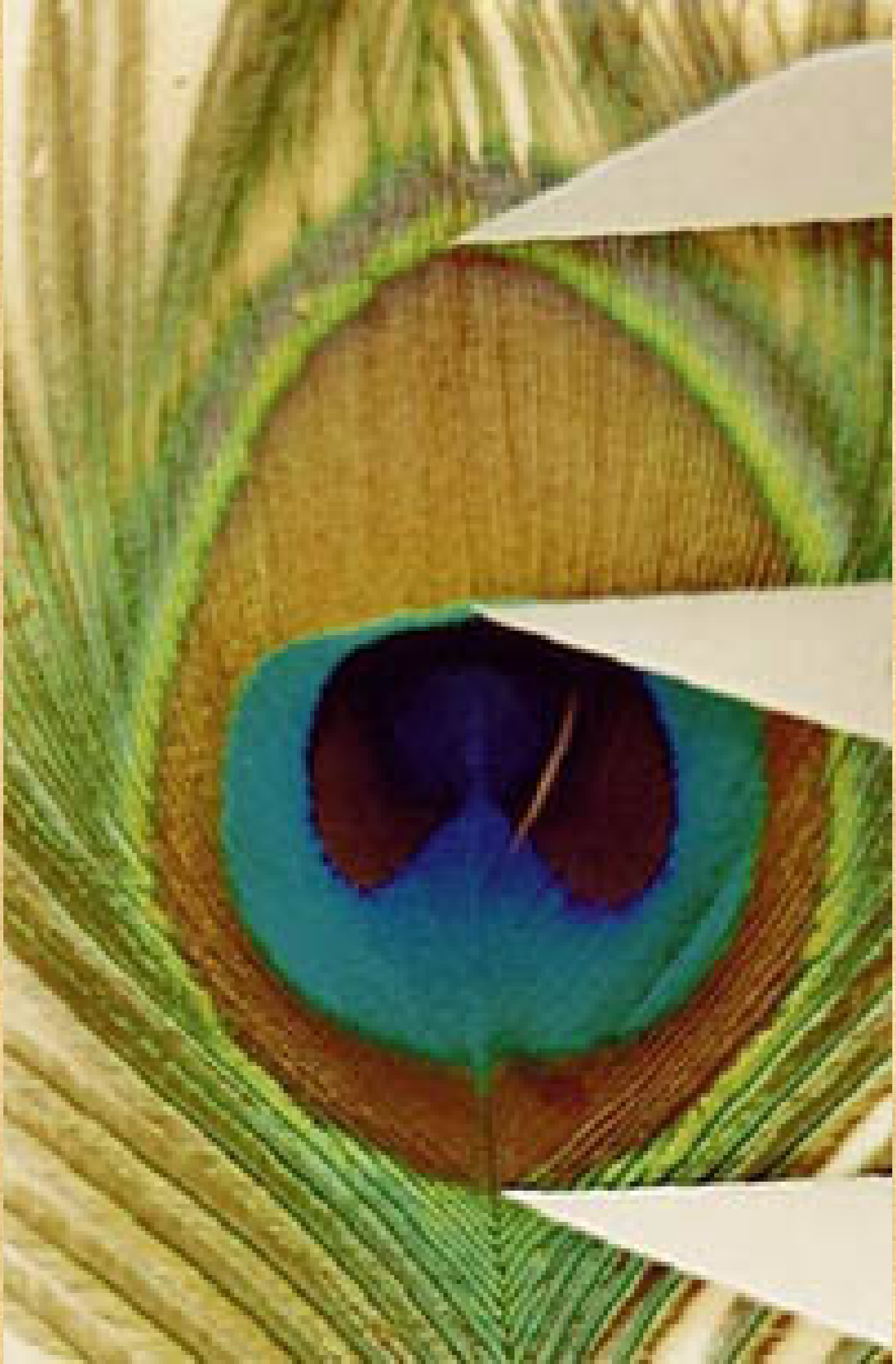
- Here you will see the Golden Ratio as it presents itself in Nature…
- Animals…
- Plants…
- See if you can identify what you are looking at.



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